Relativity of motion in vacuum

Marc-Thierry Jaekel[a], Astrid Lambrecht[b] et Serge Reynaud[b]
a Laboratoire de Physique Théorique de l'ENS *, *24 rue Lhomond, F75231 Paris Cedex 05*
b Laboratoire Kastler Brossel †, *UPMC case 74, Jussieu, F75252 Paris Cedex 05*
(Contribution for VACUUM, eds E. Gunzig and S. Diner)

The principle of relativity is one of the main general laws of physics. Since it applies in particular to motion in empty space it is related to the symmetries of this space. Thoughts about this subject within the framework of classical physics have led to the theory of relativity [1]. The emergence of quantum theory has then profoundly altered our conception of empty space by forcing us to consider vacuum as the realm of quantum field fluctuations. The notion of quantum vacuum and the existence of vacuum fluctuations unavoidably lead us to reconsider the question of relativity of motion. The present article is devoted to this aim with a main line which can be formulated as follows: *"The principle of relativity of motion is directly related to symmetries of quantum vacuum"*. Keeping close to this statement, we discuss the controversial relation between vacuum and motion. We introduce the question of relativity of motion in its historical development before coming to the results obtained more recently.

Movement in space cannot be defined otherwise than *movement in vacuum*. This assertion was formulated by Leucippus and Democritus more than 2000 years ago. In the context of that time, it has to be understood as a logical necessity rather than a physical statement in the modern sense. The existence of movement is a matter of evidence which forces us to conceive a space in which movement takes place [2]. The absence of resistance to motion is a key condition in this respect and in fact constitutes the main argument for the definition of the concept of vacuum. At the same time vacuum is a positive physical reality which clearly differs from nothingness. It is in particular the reference with respect to which movement has to be identified. These properties are certainly paradoxical at first sight. This paradox has raised numerous discussions and it has conserved its whole pertinence ever since the birth of modern physics until today.

After a long eclipse dominated by Aristotelian concepts, the question of relativity of motion was brought anew to the fore by Galileo [3]. His central argument, "*Movement is like nothing*", not only signifies that a motion with uniform velocity is indistinguishable from rest. It also implies that a motion with uniform velocity has no observable effect when it is composed with a second motion. This is the deep meaning of the famous discussion of motion in a moving boat used by Galileo in the *Dialogo* to emphasize the property of relativity [4]. Of course this property is rigorously true only when the resistance of air to motion can be ignored. Newton stressed this fact in the *Principia* in order to identify the empty space in which motion takes place with the vacuum which had just been the subject of the first experiments of Torricelli, Pascal, von Guericke and Boyle. As we know today, the Newtonian laws describing inertia or gravity already contain the Galilean principle of relativity, as it was named by Einstein, although Newton himself had argued for the absolute character of his space reference. The theory of relativity has freed space from this absolute character and one of the reasons for that was precisely that it was contradictory with Galilean relativity. After the advent of this theory, the relation between motion and space can be made explicit as follows: the expression of the laws of motion are symmetry properties of space or, equivalently, the symmetries of empty space imply the laws of motion.

The formulation of the theory of relativity has largely been built upon the symmetries of Maxwell's equations. Indeed the empty space of classical physics is a reference for writing not only the laws of mechanics but also the propagation of the electromagnetic field. However, the development of classical physics was based on the idealization that space can be thought as being absolutely empty. This classical idealization could not be maintained, not even as a limiting case, when it was realized that space is always filled with freely propagating radiation fields after the birth of statistical mechanics and then of quantum mechanics. Black body radiation is present at every non-zero temperature and it undoubtedly produces real mechanical effects. It exerts a pressure onto the reflecting boundaries of a surrounding cavity and produces a friction force when reflecting surfaces are moving. Those two effects are analogous to the pressure exerted by air molecules on the walls of a container as was made clear by Einstein's theory of Brownian motion [5]. It is precisely for explaining the properties of black body radiation that Planck introduced the first quantum law in 1900. At zero temperature this law describes a region of space limited by a cavity and entirely emptied out of radiation. In order to approach a practical realization of vacuum, it is not sufficient to remove all matter from this enclosure since one also has to lower the temperature down to zero to eliminate thermal radiation.

Later on in 1912, Planck was led to modify his law

*Laboratoire du CNRS associé à l'Ecole Normale Supérieure et à l'Université Paris-Sud

†Laboratoire de l'Ecole Normale Supérieure et de l'Université Pierre et Marie Curie associé au CNRS

in order to improve its agreement with known results in the classical regime reached at high temperatures. The modified law predicts that fluctuations remain present at zero temperature, from which Nernst deduced in 1916 that space is permanently filled with an electromagnetic field propagating with the speed of light [6]. The existence of these *zero-point fluctuations* or *vacuum fluctuations*, which correspond to half of the energy of one photon per electromagnetic field mode, was confirmed by the full quantum theory developed after 1925 [7]. Fluctuations thus appear as an inescapable consequence of Heisenberg's inequalities. To sum up this lesson of quantum theory, one may say that it establishes a necessary identity between the *potentiality* of movement and the *presence* of movement. As soon as space allows movement, which is of course an essential feature of space, then movement exists and in particular subsists at zero temperature. In the same way, as soon as space allows field propagation, which is equally a fundamental property of space, then fields are present. Quantum physics forbids the absence of movement as well as the absence of propagating fields. The ground state of a mechanical oscillator is defined as the state where its mechanical energy is minimal. In the same way, quantum vacuum is defined as the field state where the energy of field fluctuations is minimal.

Then naturally the question arises whether the presence of fluctuations in quantum vacuum is compatible with the principle of relativity of motion. It is this discussion which we will sum up in a non-technical manner in this article. More technical arguments as well as a list of references can be found elsewhere [8].

Some preliminary remarks should still be mentioned. Most of the time the discussions devoted to Heisenberg's inequalities stress the limits they impose on the precision of a measurement. Insisting on the notions of *uncertainty* or even of *indeterminacy* however presents the serious disadvantage that it fixes the ambition of physicists with respect to a classical description of physical phenomena, although quantum theory has been developed precisely in order to remove the deficiencies of this description. The successes of quantum theoretical description of natural phenomena plead for a quite renewed point of view. *Quantum fluctuations* are intrinsic properties of physical quantities which display their inherent quantum nature. Today's knowledge allows for a theoretical understanding and also for an experimental study of these properties. In certain experiments one can even manipulate the quantum fluctuations in order to reduce the noise they produce on a particular signal [9].

The fluctuations of the electromagnetic field which remain in the vacuum state have well known observable effects [10]. An isolated atom in vacuum interacts only with vacuum fluctuations and this interaction is responsible for the *spontaneous emission* processes during which the atom changes its internal state by falling from a higher energy level to a lower one. When fallen in the ground state, the state of lowest energy where it can no longer emit photons, the atom is still coupled to vacuum fluctuations and this coupling results in measurable effects like the *Lamb shift* of the atomic absorption frequencies. Two atoms in vacuum are coupled to the field fluctuations which produce an attractive force between them, the so-called *Van der Waals force*. This force plays a very important role in physical and chemical processes and its quantum theoretical interpretation has been studied since the first years of quantum theory. While studying this problem, Casimir discovered in 1948 that there exists also a force between two mirrors placed in quantum vacuum [11]. The vacuum fluctuations are modified by the cavity formed by the mirrors and their energy depends on their relative distance. Hence vacuum exerts a force which mutually attracts the mirrors to each other. This *Casimir force,* as it was called later on, depends only on the distance and on two fundamental constants, the speed of light c and the Planck constant $\hbar$. This is a remarkably universal feature in particular because the Casimir force is independent of the electronic charge in contrast to the Van der Waals forces. We have already noticed that any pragmatic definition of vacuum necessarily involves a region of space limited by some enclosure. The Casimir effect signifies in fact that the energy of vacuum depends on the configuration of this cavity from which it follows that its boundaries experience forces arising from radiation pressure of vacuum fluctuations. Although the Casimir force is relatively small it has been observed in several experiments [12].

Vacuum fluctuations thus play a central role in the modern description of the structure of matter. They explain numerous novel effects which have been observed without any ambiguity. Their existence is directly associated with a fundamental property of quantum physics, namely the representation of any physical quantity by an observable defined with the help of non-commuting mathematical objects. Their status nevertheless continues to raise intricate questions. One reason for this situation is the obvious fact that their representations are often incompatible with the intuitions inherited from classical physics. More fundamental reasons are related to the serious difficulties which have remained unsolved for a long time and, for some of them, still remain unsolved at the interface between the physics of vacuum fluctuations and the laws of mechanics or gravitation.

The archetype of these difficulties is the relation of vacuum energy to gravitation. The total vacuum energy, that is to say the energy summed over all field modes in their vacuum state, takes an infinite value. This implies that vacuum energy does not contribute in a standard way to gravitation since the universe would have a very different appearance otherwise. One simple way to deal with this problem is to set the vacuum energy equal to zero and, therefore, to use it as a reference for all other energies [13]. In contrast to a widely spread opinion this prescription does not allow to ignore the gravitational effects of vacuum. Indeed vacuum energy is modified in a space curved by the gravitational field. Furthermore,

even if the mean value of the vacuum energy does not contribute to gravitation its variations necessarily contribute to it. Yet, the spatial distribution of vacuum energy changes continuously as it is composed of field fluctuations which propagate with the speed of light. The energy density of vacuum shows fluctuations which manifest themselves as fluctuations of space-time curvature. It is possible to discuss some of these questions in analogy with the standard formalism of quantum field theory (a discussion and references can be found in [14]). However a final answer to these questions will not be reached until a satisfactory quantum description of gravitation is available.

As already stated in the introduction, from the mere point of view of logics movement in space must be understood as movement in vacuum, so that the presence of field fluctuations in quantum vacuum forces us to reconsider the notion of motion itself. This question has been discussed at length in connection with attempts to obtain a consistent description of motion for elementary quantum objects like the electron [15]. It has been learned from classical electrodynamics that the expression of the force acting on a moving charged particle contains a contribution, known as the Abraham-Lorentz force, describing the reaction to motion entailed by the electromagnetic field emitted by the particle. The modern quantum theory tells us that the radiation reaction force is directly related to the vacuum field fluctuations through the fluctuation-dissipation relations, which are the quantum generalizations of the classical results of Brownian motion theory [16]. The occurence of a dissipative force induced by motion of the electron and its direct association with vacuum fluctuations forbid to obtain any consistent description of atoms within the classical framework. In the quantum theory, this crisis was only solved at a very high price since any mechanical description of movement was abandoned for elementary quantum objects [17]. Before discussing this point in more detail in the next paragraphs, let us notice that this was not the end of the story. Vacuum field fluctuations were also shown to modify the inertial mass of a point-like scatterer and, furthermore, to lead to an infinite mass correction. A renormalization prescription was designed, which states that the *real* mass of the particle is finite, as the result of adding the infinite positive correction to a *bare* mass which is itself infinitely negative. However, other difficulties emerge as outcomes of this approach. In particular, the mechanical response of the particle to an applied force shows instability and violates causality [18]. In fact, the renormalization procedure used to keep the particle mass finite in spite of an infinite correction is incompatible with a causal mechanical description [19].

These difficulties have led most of the theoretical physicists to adopt a pragmatic point of view which has proved itself useful in the microscopic domain. Any mechanical description is then given up in this domain where physical descriptions are instead built on quantum field theory. Taken seriously, this approach forces physicists to renounce to general principles of mechanics at the elementary level of microscopic physics whereas the mechanical description of nature has a validity restricted to the macroscopic domain [20]. This *a priori* separation between microscopic and macroscopic domains has now lost the pragmatical pertinence it had when the argument was formulated, due to the progress towards highly sensitive measurements approaching the quantum level of sensitivity for macroscopic objects [21]. From a more fundamental point of view, this solution cannot be satisfactory since it rejects the mechanical questions which concern in particular inertia and gravitation outside the framework of quantum theory. Although the problems of inertia and gravitation differ, they are directly related to each other. In the domain of gravitation also, vacuum field fluctuations modify the laws of motion by the introduction of dissipative reaction terms which lead to stability problems [22]. Both cases point at difficulties generated by the mixture of classical and quantum theoretical descriptions, namely Einstein or Newton equations on one hand and quantum field theory on the other hand. It is known more generally that any description built on a too crude mixing between classical and quantum formalisms is necessarily plagued with inconsistencies [23].

The general problem of quantum field theory with moving boundaries has emerged as an outcome of the thoughts about possible approaches to this problem [24]. This subject may be considered as an extension of the study of Casimir force, precisely of vacuum radiation pressure on reflecting boundaries, to the case of moving boundaries. At the same time, it is directly related to the questions raised by quantum descriptions of motion, inertia or gravitation. As it will become apparent in the following, this problem has been given formulations well adapted to the present theoretical framework of quantum physics and has led to satisfactory solutions of some of the general difficulties mentioned above.

Let us first consider the simple case of a mirror moving with a uniform velocity. Clearly there exist two very different situations depending on whether motion takes place in vacuum or in a thermal field. The principle of relativity of motion applies to the first situation but not to the second one. Should vacuum exert a force on a mirror with a uniform velocity, the reaction of vacuum would distinguish between inertial motion and rest. As expected, quantum theory predicts the friction force to vanish in this case so that the principle of relativity of uniform motion is valid in quantum vacuum. Besides, quantum theory gives an interesting interpretation of this property according to which vacuum fluctuations appear exactly identical to an inertial observer and to an observer at rest. The invariance of vacuum under Lorentz transformations is an essential condition for the principle of relativity of motion and it establishes a precise relation between this principle and a symmetry of vacuum.

We now come to the general case of arbitrary motion in vacuum. In 1976, Fulling and Davies showed that a perfectly reflecting mirror moving in vacuum emits radi-

ation as soon as the mirror has a non uniform acceleration [25]. Meanwhile, the motion tends to be reduced to a uniform acceleration by the reaction of vacuum. The change of mechanical energy associated with damping is dissipated in form of emitted radiation and the reaction force follows from the associated momentum exchange. This *motional force* is directly related to fluctuations of the vacuum radiation pressure as they may be evaluated for a mirror at rest, in full consistency with the quantum fluctuation-dissipation relations [26,27]. When the equations of motion in vacuum are modified in order to take the motional force into account, it turns out that mirrors possess causal mechanical response functions and hence stable motions in vacuum. It has to be noted that the expression of the reaction force coincides with the Abraham-Lorentz derivative in the limit of a perfectly reflecting mirror which thus raises the same problems as in the case of the electron. This difficulty is solved by a careful description of the reflection properties of the mirror, accounting in particular for the fact that any real mirror is certainly transparent at high frequencies. The motion of a real mirror in vacuum is described in a consistent manner once these points are satisfactorily dealt with [28]. Furthermore, a full treatment of the coupling between the mirror's motion and the field scattering shows that the vacuum fluctuations of the quantum field are eventually transmitted to the position of the mirror, even if the latter was originally considered as classical. In any realistic situation, the mechanical effect of vacuum upon the mirror may be considered as vanishingly small and the mirror is found in this limit to obey the standard Schrödinger equation [29]. These results prove that the objections against the possibility of giving a consistent quantum representation of motion in vacuum can be bypassed. They also show that the solution involves subtle correlations between the descriptions of motion in vacuum and of vacuum itself. For instance, the fluctuations associated with motion in vacuum can in no way be treated as uncorrelated with the fields fluctuations of vacuum in which motion takes place.

The existence of motional effects also questions the principle of relativity of motion applied to arbitrary motions in vacuum. As the reaction of vacuum vanishes for uniform velocities it does not raise any problem to the theory of special relativity. In contrast a non-uniformly accelerated motion produces observable effects, namely the resistance of vacuum against motion and the emission of radiation by the moving mirror into vacuum. Quantum theory thus allows us to sketch the outline of a framework where the questions raised in the introduction find consistent answers. Space is not empty since vacuum fluctuations are always present. These fluctuations effectively represent a reference for the definition of motion because they give rise to real dissipative effects in the general case of an arbitrary motion. At the same time, they do not damp uniform motions since quantum vacuum obeys Lorentz invariance. The physical properties of quantum vacuum are thus consistent with logical requirements of ancient atomists as well as with the Galilean principle of relativity of motion.

In this context, it would be extremely interesting to obtain experimental evidence for motional dissipative effects. Although these effects are extremely small for any motion which could be achieved in practice for a single mirror, an experimental observation could turn out to be conceivable with a cavity, instead of a single mirror, oscillating in vacuum. Due to a resonance enhancement of the emission of motional radiation the experimental figures become indeed much more promising in this case [30].

The radiation reaction force calculated by Fulling and Davies for a single mirror in vacuum is proportional to the Abraham-Lorentz derivative associated with its motion. Hence, it vanishes exactly for a uniformly accelerated motion. One question then arises, which naturally generalizes the one already met when considering uniform motion. How do vacuum fluctuations appear for an observer with uniform acceleration? The absence of reaction of vacuum against uniformly accelerated motion would suit well the invariance of vacuum fluctuations under a group of transformations corresponding to observers with uniform acceleration as well as uniform velocity. This question has raised and still raises numerous controversial discussions between physicists [31]. To understand this controversy it is necessary to describe what is often called the *Unruh effect* which predicts that vacuum fluctuations should appear as thermal fluctuations in a uniformly accelerated reference frame [32]. The effective temperature, which is proportional to Planck constant and to acceleration, is analogous to the Hawking temperature of the field radiated by the surface of a black hole [33]. This analogy plays an important role in the arguments pleading in favor of the physical reality of the Unruh effect but it can be questioned as both situations are clearly different within the framework of general relativity. The curvature of space-time is not involved in a change of reference frame whereas it plays a central role in the description of a black hole.

Concerning the problem of relativity of motion, the Unruh effect shows serious drawbacks. If vacuum is really different for accelerated and inertial observers, this certainly makes it difficult, if not impossible, to describe inertia in a consistent quantum formalism. In particular difficulties should arise when analysing situations which involve composed motions. For example a motion with uniform velocity in an accelerated frame should give rise to a vacuum reaction like uniform velocity in an ordinary thermal field whereas a uniformly accelerated motion in a frame obtained from a Lorentz transformation would be free from dissipation. Contradictions are also met concerning energy conservation for an accelerated detector. Furthermore, the interpretation of detection processes appear to depend on the reference frame. As far as this point is concerned different results have been obtained depending on the model used for the detector [31]. For those models involving detectors which click

when accelerated in vacuum it can even happen that the detection process is found to be related now to absorption then to emission of a photon. More generally it is impossible to preserve the usual understanding of the principle of relativity because the notions of vacuum or photon number are not the same for different observers [34].

These difficulties have their origin in a too rapid identification of uniformly accelerated reference frames with their Rindler representation. This particular representation of accelerated frames favors a criterion of rigidity in the transformation of solid bodies. A consequence of this choice is that Maxwell equations are modified in the change of reference frame. The Unruh effect essentially tells us that the definition of vacuum is also altered in a Rindler transformation and that the accelerated vacuum obtained after the transformation appears as a thermal field. But the Rindler representation of accelerated reference frames is not the only possible one. In fact an infinite number of representations exist when one only imposes the condition that a given point with uniform acceleration is brought to rest. Furthermore, the theory of general relativity does not provide any manner to privilegiate one particular representation of accelerated frames since it convincingly argues that physical results cannot depend on the choice of a particular map of space-time.

In quantum theory in contrast, motion must be understood as taking place in a space filled with vacuum fluctuations. This feature implies to attach a particular importance to those transformations to accelerated frames which leave vacuum invariant. This leads to favor conformal coordinate transformations which have been known for a long time to leave Maxwell equations invariant [35]. These tranformations represent a natural extension of Lorentz transformations which also include uniformly accelerated frames [36]. An essential property of conformal transformations is to preserve the definition of vacuum fluctuations [37] and, more generally, the definition of particle number [38]. Hence there is no more difference between the viewpoints of accelerated and of inertial observers as far as their perception of vacuum is concerned, provided that accelerated observers are defined through conformal transformations. The fact that vacuum exerts no force on a uniformly accelerated object, already discussed for electrons and for mirrors, is in fact a direct consequence of this symmetry property of quantum vacuum. In this sense, the principle of relativity which was known for uniform velocities now has a domain of application extended to uniform accelerations.

In the conformal representation of accelerated frames, the Unruh effect disappears as well as the paradoxes it creates. A deep reason for the difficulties mentioned above is that Rindler transformations do not form a group. In fact they do not even compose well with Poincaré transformations. But the discussion of the principle of relativity is mainly based on group properties obeyed by the composition of motions and these arguments do not hold for Rindler transformations. The situation is much more favorable for conformal coordinate transformations which form a group including the Poincaré transformations besides the conformal changes to accelerated frames. In particular, the composition of a uniformly accelerated motion with an inertial motion results in another uniformly accelerated motion. Associated with preservation of Maxwell equations and of electromagnetic vacuum, these group properties amply justify the use of conformal transformations to represent uniformly accelerated reference frames. The conformal representation is the natural choice when relativistic properties are interpreted as invariance properties rather than mere form-invariance relations [39].

Furthermore, a clear understanding of the notion of quantum particles requires that the notion of vacuum has been understood before. This point has been thoroughly discussed in the present paper for what concerns the principle of relativity of motion. It must equally be taken into account when considering the extension of the equivalence principle to the quantum domain. The thoughts presented here plead for bringing out the symmetries of vacuum, the quantum version of empty space, in the forefront of primary questions. As it has been shown here, this should ensure compatibility of the conception of motion with the principle of relativity. It has also been shown elsewhere that this approach fits perfectly well the description of localization in space-time built upon symmetries of quantum fields. Precisely, conformal symmetry allows to define quantum observables associated with positions in space-time and to obtain their transformations under Lorentz transformations and transformations to accelerated frames [40].

It has often been noted that the serious problems arising in the mechanical description of electrons is directly associated with the fact that they are treated as point-like structures although such a treatment certainly contradicts their quantum nature. More generally, the classical conception of space as a set of points upon which classical relativity is built with the help of differential geometry is challenged by the quantum nature of physical observables. The design of new conceptions of motion and localization in space could therefore reveal itself a preliminary step to the solution of yet unsolved problems lying at the interface between quantum theory and gravitation.

---

[1] Einstein A. 1946 *The Meaning of Relativity* (Princeton University Press)

[2] Russell B. 1961 *History of Western Philosophy* (George Allen & Unwin; reprinted by Routledge 1991)

[3] Koyré A. 1957 *From the Closed World to the Open Universe* (JohnHopkins Press)

[4] Balibar F. 1984 *Galilée, Newton lus par Einstein Espace et relativité* (Presses Universitaires de France); 1992 *Einstein 1905 De l'éther aux quanta* (Presses Universitaires de France)

[5] Einstein A. 1905 *Ann. Physik* **17** 549; 1909 *Phys. Z.* **10** 185; 1917 *Phys. Z.* **18** 121

[6] Planck M. 1900 *Verh. Deutsch. Phys. Ges.* **2** 237; 1911 *ibid.* **13** 138; Nernst W. 1916 *ibid.* **18** 83

[7] Dirac P. A. M. 1958 *The Principles of Quantum Mechanics* (Oxford University Press)

[8] Jaekel M. T. and Reynaud S. 1997 *Rep. Progr. Phys.* **60** 863

[9] Reynaud S., Heidmann A., Giacobino E. and Fabre C. 1992 in *Progress in Optics XXX* ed. E. Wolf (North Holland) 1; Giacobino E. and Fabre C. (eds) 1992 *Quantum Noise Reduction in Optical Systems Applied Physics* **B 55** 189-303 (Special Issue); Reynaud S., Giacobino E. and Zinn-Justin J. (eds) 1997 *Quantum Fluctuations, Proceedings of Les Houches Summer School LXIII* (Elsevier)

[10] Cohen-Tannoudji C., Dupont-Roc J. and Grynberg G. 1988 *Processus d'interaction entre photons et atomes* (InterEditions) [1992 *Atom-Photon Interactions* (Wiley)]; Sciama D. W. 1991 in *The Philosophy of Vacuum* eds S. Saunders and H. R. Brown (Clarendon)

[11] Casimir H. B. G. 1948 *Proc. K. Ned. Akad. Wet.* **51** 793

[12] Deriagin B. V. and Abrikosova I. I. 1957 *Jh. Eksp. Teor. Fis.* **30** 993 [*Soviet Physics JETP* **3** 819]; Spaarnay M. J. 1958 *Physica* **XXIV**, 751; Tabor D. and Winterton R. H. S. 1968 *Nature* **219**, 1120; Black W., De Jong J. G. V., Overbeek J. Th. G. and Sparnaay M. J.1968 *Trans. Faraday Soc.* **56** 1597; Sabisky E. S. and Anderson C. H. 1973 *Phys. Rev.* **A7** 790; Lamoreaux S. K. 1997 *Phys. Rev. Lett.* **78**, 5

[13] Enz C. P. 1974 in *Physical Reality and Mathematical Description* eds C. P. Enz and J. Mehra (Reidel)

[14] Jaekel M. T. and Reynaud S. 1995 *Ann. Physik* **4** 68

[15] Dirac P. A. M. 1938 *Proc. Roy. Soc. London* **CLXVII A** 148

[16] Callen H. B. and Welton T. A. 1951 *Phys. Rev.* **83** 34; Kubo R. 1966 *Rep. Prog. Phys.* **29** 255

[17] Heisenberg W. 1930 *The Physical Principles of the Quantum Theory* (Dover)

[18] Rohrlich F. 1965 *Classical Charged Particles* (Addison Wesley)

[19] Dekker H. 1985 *Phys. Lett.* **A 107** 255; 1985 *Physica* **133 A** 1

[20] Rosenfeld L. 1963 *Nucl. Phys.* **40** 353

[21] Braginsky V. B. and Khalili F. Ya. 1992 *Quantum Measurement* (Cambridge University Press); Bocko M. F. and Onofrio R. 1996 *Rev. Mod. Phys.* **68** 755

[22] Fulling S. A. 1989 *Aspects of Quantum Field Theory in Curved Spacetime* (Cambridge University Press)

[23] de Witt B. S. 1962 *J. Math. Phys.* **3** 619

[24] de Witt B. S. 1975 *Phys. Rep.* **19** 295

[25] Fulling S. A. and Davies P. C. W. 1976 *Proc. R. Soc.* **A 348** 393

[26] Jaekel M. T. and Reynaud S. 1992 *Quantum Opt.* **4** 39

[27] Barton G. 1994 *Cavity Quantum Electrodynamics (Supplement: Advances in Atomic, Molecular and Optical Physics)* ed. P. Berman (Academic Press)

[28] Jaekel M. T. and Reynaud S. 1992 *Phys. Lett.* **A 167** 227

[29] Jaekel M. T. and Reynaud S. 1993 *J. Physique* **I 3** 1

[30] Lambrecht A., Jaekel M. T. and Reynaud S. 1996 *Phys. Rev. Lett.* **77** 615

[31] Unruh W. G. and Wald R. M. 1984 *Phys. Rev.* **D 29** 1047; Unruh W. G. 1992 *Phys. Rev.* **D 46** 3271; Grove P. G. 1986 *Class. Quantum Grav.* **3** 793; 1988 *Class. Quantum Grav.* **5** 1381; Raine D. J., Sciama D. W. and Grove P. G. 1991 *Proc. R. Soc. London* **A 435** 205

[32] Davies P. C. W. 1975 *J. Physics* **A 8** 609; Unruh W. G. 1976 *Phys. Rev.* **D 14** 870; Birrell N. D. and Davies P. C. W. 1982 *Quantum Fields in Curved Space* (Cambridge University Press)

[33] Hawking S. W. 1975 *Comm. Math. Phys.* **43** 199

[34] Ginzburg V. L. and Frolov V. P. 1987 *Usp. Fiz. Nauk* **153** 633 [*Sov. Phys. Usp.* **30** 1073]

[35] Bateman H. 1909 *Proc. London Math. Soc.* **7** 70; *ibid.* **8** 223; Cunningham E. 1909 *ibid.* **8** 77

[36] Fulton T., Rohrlich F. and Witten L. 1962 *Rev. Mod. Phys.* **34** 442; 1962 *Nuovo Cimento* **26** 653

[37] Jaekel M. T. and Reynaud S. 1995 *Quantum Semiclass. Opt.* **7** 499

[38] Jaekel M. T. and Reynaud S. 1995 *Brazilian J. Phys.* **25** 325

[39] Norton J. D. 1993 *Rep. Progr. Phys.* **56** 791

[40] Jaekel M. T. and Reynaud S. 1996 *Phys. Rev. Lett.* **76** 2407; *Phys. Lett.* **A220** 10; 1997 *EuroPhys. Lett.* **38** 1; 1998 *Foundations of Physics* to appear